PRINCETON COS CON

Problem 2: Lord of the Lanternflies [Email Submission]

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1 Background: Cellular Automata

(*This part is just for fun. It's still pretty interesting background thought; would totally recommend!*) Back in the 1940s, two first-rate geniuses at Los Alamos National Laboratory were studying two seemingly different problems. John Von Neumann — a boyish Hungarian-American commonly regarded to be one of the most recent truly great mathematicians — was thinking about how to make a robot create itself; Stanislaw Ulam — a burly Pole who had spent a lot of time toying with thermonuclear weapons — was trying to study how crystals grow. As it would turn out, these two problems are related, because these systems can be both modelled with *cellular automata*!

A cellular automaton is a model of computation that can be thought of as just a (potentially infinite) grid where each cell has one of finitely many states. The automaton has a set of rules that determines how each cell will evolve after one time step. One of the simplest cellular automata is Conway's Game of Life, where the state of one cell only depends on its direct neighbors (see the figure below; pretty cool, huh?). In the past fifty years, people have found that cellular automata can be applied to a variety of scientific questions, including in epidemiology, demography, ecology, or just any application which wants to model some autonomous agent's growth and/or decay over time. In this problem, you'll be trying to prove something about a very simple cellular automaton — note that in general, it's extremely hard, maybe even impossible, to prove things about arbitrarily complicated cellular automata!

2 Problem Statement

Despite the best efforts of Princetonians to stomp them out, lanternflies continue to swarm the campus. Soon it may be too late to ever remove them! You talk to one of your EEB friends who tells you that they have figured out how lanternflies breed. Specifically, lanternflies live on a square grid with dimensions $n \times n$ — and if a square (i, j) is empty, a new lanternfly will spawn there on the next day if and only if there is a lanternfly on at least two out of the following four squares:

$$(i-1, j), (i+1, j), (i, j-1), (i, j+1)$$

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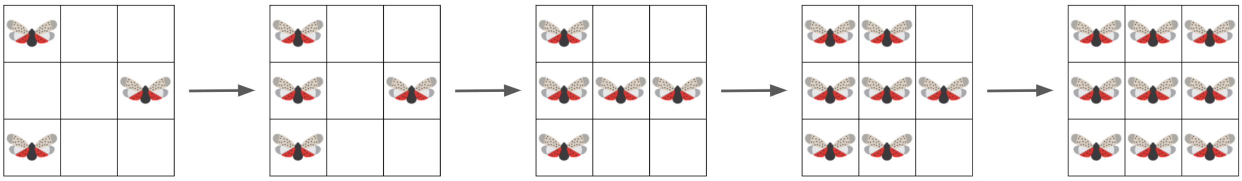
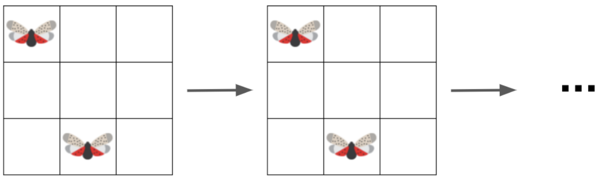
In other words, in at least two of the immediately adjacent squares in the cardinal direction. Once born, they never leave their square...

You are curious what the minimum number of lanternflies that need to exist on the grid in order for them to eventually take over the whole grid if no one stomps them out. Find this value in terms of n (there is an exact formula!). *Note you will need to both prove that an arrangement exists of $f(n)$ lanternflies that eventually takes over the whole grid and that no arrangement of less than $f(n)$ lanternflies could ever take over the whole grid.*

Example: Configuration that Eventually Takes Over Full Grid



Example: Grids with No Births



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