Problem 1: Fairly Filtering Freshmen [HackerRank]

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This problem is a re-wording of the Necklace Problem. Click here to see a video of Princeton professor Noga Alon explaining it.

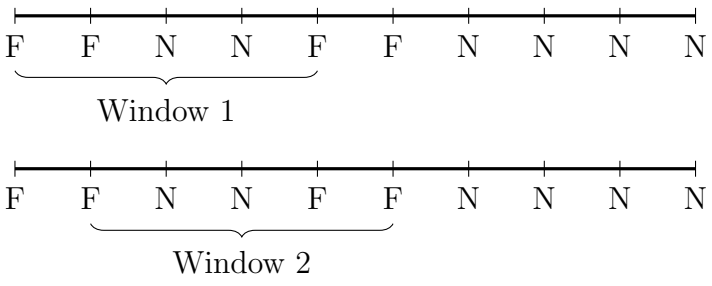
We start with the basics. If either the number of freshmen or the number of students is odd, you can print -1 since you cannot evenly split the students among the two clubs. For the rest of the solution, assume that we have an even number of freshmen and an even number of students. The main idea in this problem is that you never need more than 3 segments. Let us try to see why.

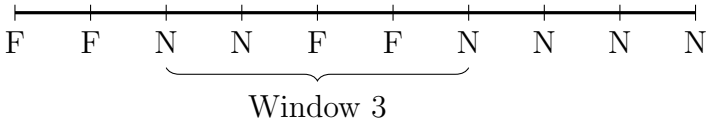
First of all, we introduce some variables: let `N` be the total number of students and let `F` be the total number of freshmen, both of which are even.

When can you make an assignment using just two segments? In order to have just two segments, these segments must be the left and right halves of the input string (since they need to have an equal number of students). You can do this if and only if the left and right half halves have `F`$/2$ freshmen. You can check for this in $\mathcal{O}(N)$ time.

Now assume that the left and right halves have a different number of freshmen. And assume, for now, that the left half has fewer than `F`$/2$ freshmen, which means that the right half has more than `F`$/2$ freshmen.

Now consider what is usually called a "sliding window" across the input string. This sliding window is a substring of length `N`$/2$. The left half of the string is an example of such a substring. We can keep sliding this window to the right, shifting it by 1, until we get to the right half of the string. This gives us a total of `N`$/2+1$ possibilities for the location of the window.





We further notice that sliding the window to the right changes the number of freshmen by at most 1. This is because you can gain at most one freshmen from the right and lose at most one freshmen from the left. Since the first window has fewer than `F`/2 freshmen and the last window has more than `F`/2 freshmen, and the number of freshmen always changes by at most 1, a discretized version of the Intermediate Value Theorem tells you that there must exist a window with exactly `F`/2 freshmen.

Now that you have this window, you can send all the students in this window to Cannon and the ones to the left and right of this window to Tower, giving you exactly three segments. The case where the left half of the input string has more than `F`/2 freshman looks identical.

Notice that as you keep sliding the window, you can compute the number of freshmen in a window in constant time by just looking at the first element of the previous window and the last element of the current window. Thus, you can do this in $\mathcal{O}(N)$ time.

Plaudits:

- Congratulations to Seyoon Ragavan '21 and Lucas Salvador '20 (now grad) for being the first to solve this problem! They did it in an impressive 36 minutes.
- Congratulations to Nivedhitha Sivakumar '23 and Andrew Alexander '23 for being the first undergraduate team to solve the problem! They did it in an also impressive 68 minutes.